

Practice Test J: Answer Key and Rubric

Tutor copy. Every answer was checked by symbolic computation (SymPy). All problems are original; see the notes file for provenance. Prepared with assistance from Claude, an AI model made by Anthropic.

General grading rules

- **Answer-only items** (Part I, Part II, Q13, Q16): all or nothing.
- **Infinite limits.** The instruction box tells students to write ∞ or $-\infty$. “DNE” for a limit whose answer is ∞ or $-\infty$ earns 1 point of the item’s value. A wrong sign earns 0. Where one side is ∞ and the other $-\infty$, the answer is DNE and “ ∞ ” earns 0.
- **Function-value items** (Q1 value parts, Q2(c), Q7(e), Q8, Q13): writing the *limit* where the function value was asked earns 0. “Undefined,” “not defined,” or “does not exist” are all accepted for an undefined value.
- **Show-work items** (Parts III–V): a bare correct answer earns at most half credit.
- **Method is not graded for efficiency.** A correct answer reached by long but valid algebra gets full credit. The time it costs is the penalty.

Test J

Item	Answer	Rubric / trap
1(a)	$-\infty$	$\lim_{x \rightarrow -2} f(x)$. Both sides go the same way. <i>Trap</i> : reflexive “DNE” (1 pt).
1(b)	2	$f(4)$. <i>Trap</i> : function value; read the closed dot (or its absence), not the curve.
1(c)	4	$\lim_{x \rightarrow 4} f(x)$.
1(d)	$-\infty$	$\lim_{x \rightarrow -2^-} f(x)$.
1(e)	-2	$\lim_{x \rightarrow 1^+} f(x)$.
1(f)	DNE	$\lim_{x \rightarrow 1} f(x)$. One-sided limits differ.
1(g)	undefined	$f(-2)$. <i>Trap</i> : function value; read the closed dot (or its absence), not the curve.
2(a)	4	<i>Trap</i> : “+” means from the right: the right half of the table.
2(b)	4	
2(c)	8	Function value read directly from the middle column.
3	$-\frac{1}{6}$	Direct substitution; 2 setup, 3 value. <i>Trap</i> : the shared factor $(x - 1)$ pulls students into factoring, which also works but costs time.
4	$\frac{1}{3}$	Direct substitution. “0/0,” “undefined,” “DNE”: 0 pts. Rationalizing is valid but slow.
5	-9	1 pt notes 0/0; 3 pts synthetic division by -3 on 1, 2, 3, 18 giving $x^2 - x + 6$, and denominator $(x + 3)(x + 1)$; 2 pts value.
6	(a) ∞ (b) $-\infty$ (c) DNE	3 each; sign reasoning required for full credit.
7	(a) 20 (b) 6 (c) 8 (d) DNE (e) 6	2 each. <i>Trap in (a)</i> : $x = 6$ uses the second piece; the first piece gives 38. <i>Trap in (e)</i> : function value.
8	undefined	$\frac{0}{0}$. <i>Trap</i> : simplifying gives the limit 12; that answer earns 0.
9	Continuous	(1) $f(2) = 0$; (2) $\lim_{x \rightarrow 2} f(x) = 0$; (3) equal. 2 pts per condition.

Item	Answer	Rubric / trap
10	Not continuous	(1) $h(1) = -2$; (2) left 1, right -2 , so the limit does not exist; (3) fails. 2 pts per condition.
11	No	At the endpoint $x = 1$ the definition needs continuity from the left: $\lim_{x \rightarrow 1^-} f(x) = 2$ and $f(1) = -2$, which differ. The curve is unbroken inside the interval; the failure is at the endpoint. 2 answer, 2 reason.
12	(a) Yes (b) No	Denominator zero at $-3, 0$. Each part: 1 answer, 2 reason. <i>Trap</i> : the numerator zero $x = 1$ is irrelevant; watch closed versus open brackets.
13	0	<i>Trap</i> : IVT heading; only a value is asked. Watch signs of negative inputs.
14	Yes; $c = 3$	2 continuity (polynomial); 2 $f(1) = 6$, $f(4) = 36$, and 24 is between; 2 solve $f(x) = 24$: $x = 3$ or $x = -8$, keep 3 only.
15	No	s has a jump at $x = 3$, so the IVT does not apply. <i>Trap</i> : the graph does reach 1; “Yes” earns 0 because the question asks what the theorem guarantees. 2 answer, 2 reason.
16(a)	F	False. The IVT gives <i>at least</i> one; a continuous zigzag can cross 0 three times. <i>Trap</i> : the word “exactly.”
16(b)	T	True. Continuous, with a negative value at one end and a positive value at the other.

Points: $14 + 6 + 5 + 5 + 6 + 9 + 10 + 3 + 6 + 6 + 4 + 6 + 2 + 6 + 4 + 8 = 100$.

Interval notation in IVT items. Q16(a) and Q16(b) use a closed interval $[a, b]$ for c ; Q14 and Q15 use the open interval (a, b) . The mix is deliberate. In every item the target value differs from both endpoint values, so no endpoint can be a solution and the choice of bracket never changes an answer.